JOURNAL OF COMPUTATIONAL PHYSICS **135,** 101–102 (1997)
ARTICLE NO. CP975722

Introduction to ''Computational Design for Long-Term Numerical Integration of the Equations of Fluid Motion: Two-Dimensional Incompressible Flow. Part I''

In the late 1950s and early 1960s a small group of mostly young meteorologists in a few institutions started what later became known as large eddy simulation. Their work initially grew out of the first successful numerical weather prediction experiments, led by John von Neumann in the late 1940s. The group was small partly because numerical simulation of fluid dynamics was looked on with skepticism, but mostly because only a few computers capable of making a start on the problems existed, nearly all of which were in the United States. The first important published work was by Phillips [2], who generated a model of the atmospheric general circulation from nearly biblical foundations—''In the beginning there was darkness and void, and God sent forth the light.'' An early followup was an attempt to simulate buoyant convection by Malkus and Witt [1]. Both these efforts, though successful to a point, were marred by a mysterious form of computational instability, identified by Phillips [3] as associated with aliased products of variable quantities, one of them usually flow velocity. Phillips overcame this ''nonlinear instability'' by Fourier filtering out the high-frequency modes (last octave), thus anticipating, though not quite correctly, the ''2/3 rule'' later used in pseudo-spectral models. Incidentally, I believe this occurred before the ''fast Fourier transform'' was widely known and applied, although for the low resolutions available at that time it probably would not have mattered much.

It was generally thought that nonlinear instability was an inherent limitation of centered difference schemes through their wavenumber aliasing. Akio Arakawa realized, however, that if a quadratic variable, like kinetic energy, could be conserved in a way similar to its conservation in the continuous fluid equations, a degree of stability was assured. In the early 1960s he began showing, informally and through seminars and conference papers, a possible remedy for the problem, using second order numerical approximations to the governing equations which allowed conservation of quadratic moments. Akio was cautious and deliberate about going into print with his methods, preferring to first develop careful proofs and rather general algorithms, including those using fourth order accuracy and curvilinear coordinates. This time delay allowed, or nearly forced, some of us to take advantage of his early work and show its practicality before it was formally exhibited under his name. To our relief that exhibition finally occurred in 1966, when Akio published his paper in the new *Journal of Computational Physics.* As they say, the rest is history. Arakawa's methods, together with increasing computing capabilities, rapidly opened the field. It became clear that such schemes could be developed for almost any quantity transported by a variable flow field and with almost any gridding scheme. Admittedly there were and still are holdouts for upwind differencing schemes, which are also stable, though damped.

The *Science Citation Index* helps trace the impact of this paper. In the first two years after publication 3 and 6 citations were reported. After that the number built up from 14 in 1969 to 26 in 1990, and has generally run between 15 and 20 per year, with a total of about 500 over 30 years. Examination of the 18 references for one year, 1983, shows that half of them appeared in standard meteorological journals, a few in less well known such journals, and about one-third in journals of other fields. The usual gradual decrease in citations for classic papers is apparently countered by the continued increase in numerical simulations using Arakawa's methods. However, the citations underestimate the impact, since many recent authors who use the methods make primary reference to work of a colleague who has developed them for application to a specific field, or they simply use a generic expression, such as ''variance conserving methods.''

Although Arakawa's methods eliminated the instability problem, they remain subject to aliasing and to phase errors, often serious for short wavelengths, and they also remain subject to the Courant time step limitation. A large number of alternative approaches have been developed to try to overcome these problems, including the pseudo-spectral technique, high-accuracy monotone upwind schemes, semi-Lagrangian schemes, and others. For simple geometries, the spectral or pseudo-spectral methods with the 2/3 rule wavenumber cutoff eliminate aliasing and its associated instability and offer higher accuracy. ''Monotonic'' schemes are often applied when it is important not to exceed the maximum or minimum value of some transported quantity. Semi-Lagrangian schemes allow, under certain conditions, a longer time step. The Arakawa tech-

0021-9991/97 $25.00

niques are still widely used, however. Often the equations of motion are simulated with an Arakawa technique, while transport of a scalar is simulated with a monotonic scheme.

Perhaps because of its long gestation period, the paper was written in a somewhat introverted style using rather difficult notation, although a determined graduate student can plow through it and I am not aware of any significant errors. A new reader might be puzzled by Arakawa's emphasis on two-dimensional flows and the joint conservation of two quadratics, kinetic energy and enstrophy. This occurred partly because the early numerical prediction models and subsequent research applications were generally confined to two dimensions, and they showed some success because the large scale motions of the atmosphere are quasi-two-dimensional. Within this limitation, Arakawa's stated motive was to calculate the spectral dispersion of modes as accurately as possible by correctly simulating the ratio of energy to enstrophy. Modern simulations or predictions of meteorological evolution use fully three-dimensional models, and the use of Arakawa's technique is usually confined to numerical conservation of kinetic energy and/or the square of one or more scalar variables.

In summary, it is highly appropriate that the *Journal of Computational Physics* editors have chosen this work to help memorialize the journal's first 30 years. It is certainly one of the most influential papers of that period in meteorology and related sciences, and I am honored to have been invited to introduce it.

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